Math 335 Sample Problems

One notebook sized page of notes (*one side*)will be allowed on the test. You may work together on the sample problems – I encourage you to do that. The test will cover 4.5-4.7 and 5.3-5.8, 6.1. There may be homework problems on the test. The midterm is on Monday, February 4.

1. Suppose f is continuous on $[0, \infty)$ and $|xf(x)| < 1$ for $x \geq 1$. Prove or give a counterexample to the statement that $\int_1^\infty f(x)dx$ converges.

2. Let C be the curve of intersection of $y + z = 0$ and $x^2 + y^2 = a^2$ oriented in the counterclockwise direction when viewed from a point high on the z-axis. Use Stokes' theorem to compute the value of $\displaystyle\int_C (xz + 1)dx + (yz + 2x)dy$.

3. Let
$$\phi(x) = \int_0^\pi \cos(x \sin t)dt.$$
Prove that
$$x\phi''(x) + \phi'(x) + x\phi(x) = 0.$$

4. (a) Prove that $\displaystyle\int_C \frac{-ydx + xdy}{x^2 + y^2}$ is not independent of path on $\mathbf{R}^2 - \mathbf{0}$.

   (b) Prove that $\displaystyle\int_C \frac{xdx + ydy}{x^2 + y^2}$ is independent of path on $\mathbf{R}^2 - \mathbf{0}$. Find a function $f(x, y)$ on $\mathbf{R}^2 - \mathbf{0}$ so that $\nabla f = (\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2})$.

5. Prove that $\displaystyle\int_0^\infty \cos x^2 dx$ converges, but not absolutely.

6. Decide if the following integrals converge conditionally, converge absolutely, or diverge.

   (a)
$$\int_{-\infty}^{+\infty} x^2 e^{-|x|}dx$$

   (b)
$$\int_0^\pi \frac{dx}{(\cos x)^{\frac{2}{3}}}$$

(c)
$$\int_1^\infty \frac{\sin(1/x)}{x} dx$$

7. Let f and g be integrable on $[a, b]$ for every $b > a$.

   (a) Prove that
   $$\left(\int_a^b |fg|\right)^2 \leq \int_a^b f^2 \int_a^b g^2.$$
   You must give a proof of this. It is not proved in the text.

   (b) Prove that if $\int_a^\infty f^2$ and $\int_a^\infty g^2$ converge then $\int_a^\infty fg$ converges absolutely.

8. Let $a_n = \log(\frac{n}{n+1})$. Does $a_n \to 0$? Does the series $\sum_1^\infty a_n$ converge? If so, find its limit.

9. Let S be the surface (torus) obtained by rotating the circle $(x-2)^2 + z^2 = 1$ around the z-axis. Compute the integral $\displaystyle\int_S \mathbf{F} \cdot \mathbf{n} dA$, where $\mathbf{F} = (x + \sin(yz), y + e^{x+z}, z - x^2 \cos y)$.

10. Let $w(x)$ satisfy $w''(x) + w(x) = 0, w(0) = 0, w'(0) = 1$. Let $f(x) = \displaystyle\int_0^x (w(x-y))h(y)dy$. Prove that
$$f''(x) + f(x) = h(x), f(0) = 0, f'(0) = 0.$$

11. We have covered the following:

    (a) Surface area
    (b) Divergence theorem
    (c) Stokes' theorem
    (d) Integrating vector derivatives
    (e) Integrals dependent on a parameter
    (f) Improper single and multiple integrals
    (g) Introduction to infinite series.

12. There may be homework problems or example problems from the text on the midterm.